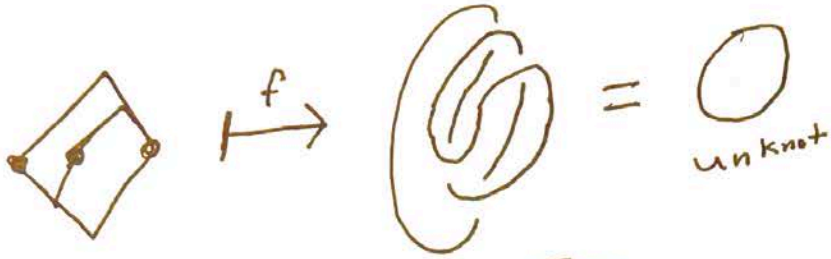
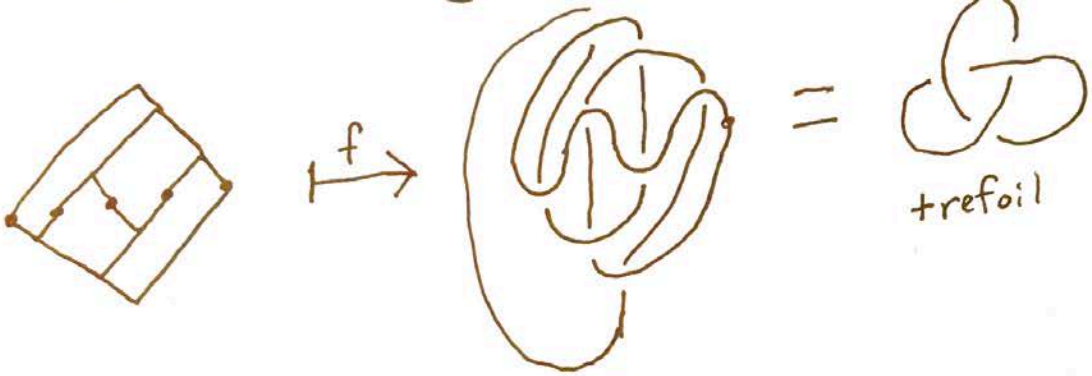
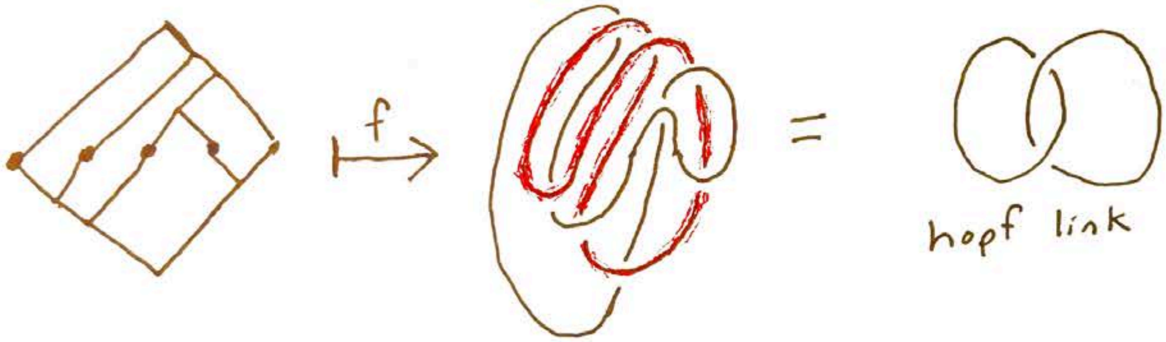
Recall our function

$f:$ Thomson's group F $\longrightarrow$ Links



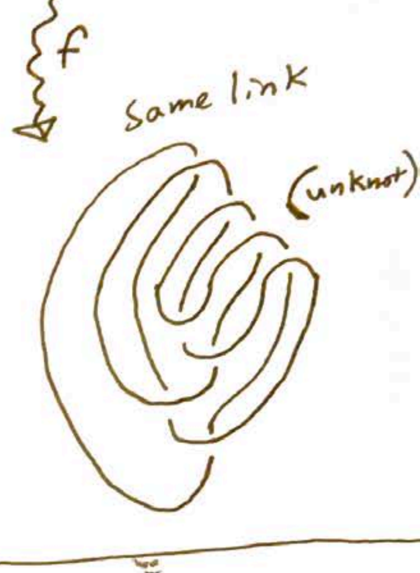




??? $\overset{f}{\longmapsto}$ Your favorite link here

Vaughan Jones shows that indeed, f is surjective.

- Analogous to Alexander's theorem about braids.
- Proof uses signed plane graphs.



Which Reidemeister moves? Algorithm:

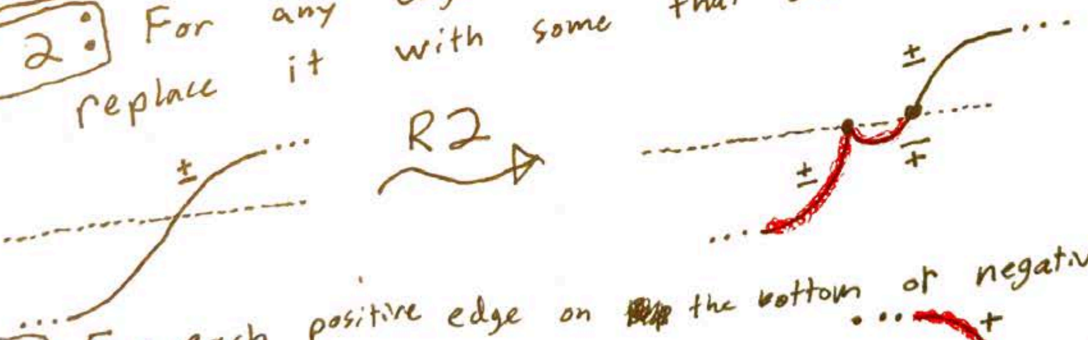
Input: Signed planar graph representing a link L

Output: Hill diagram, i.e. a signed planar graph representing L and satisfying:

- All vertices on x-axis.
- Positive edges in top half-plane, (negative " " bottom " ").
- Each half-plane's edges form a directed tree rooted at the leftmost vertex, with edges directed left to right.

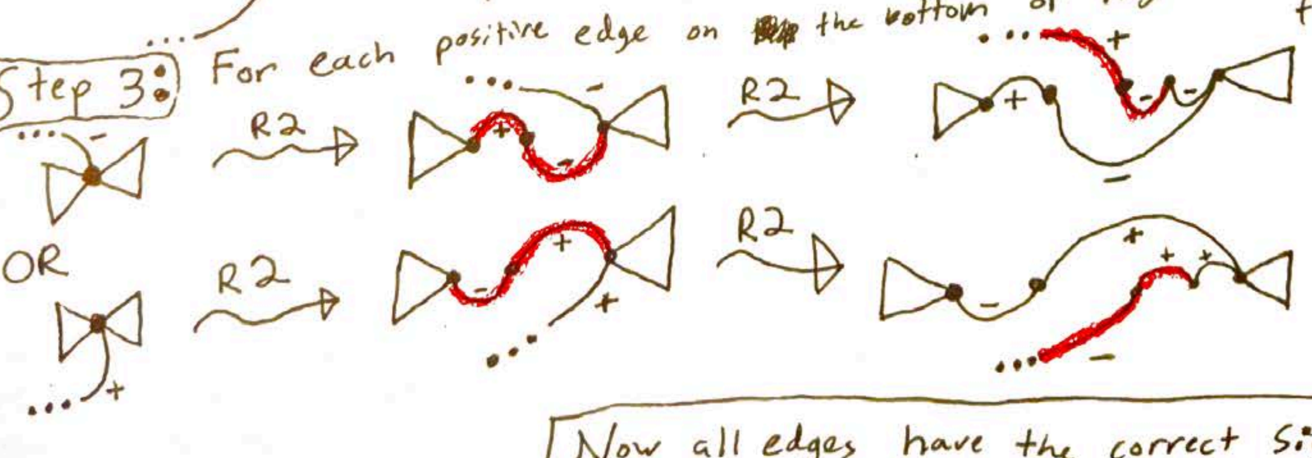
Step 1: Homotope the vertices to the x-axis.

Step 2: For any edge that crosses the x-axis, replace it with some that don't:



Now the graph can be drawn with only semi-circles.

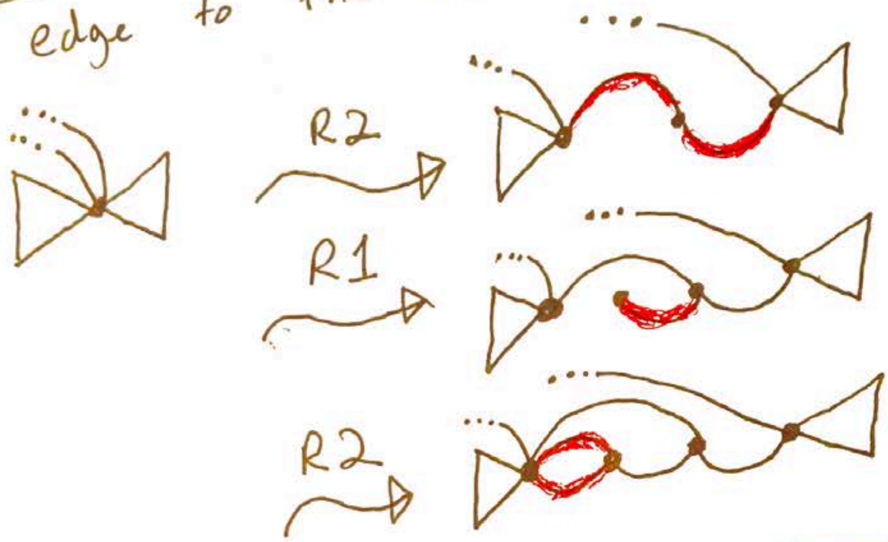
Step 3: For each positive edge on the bottom or negative edge on top, fix it:



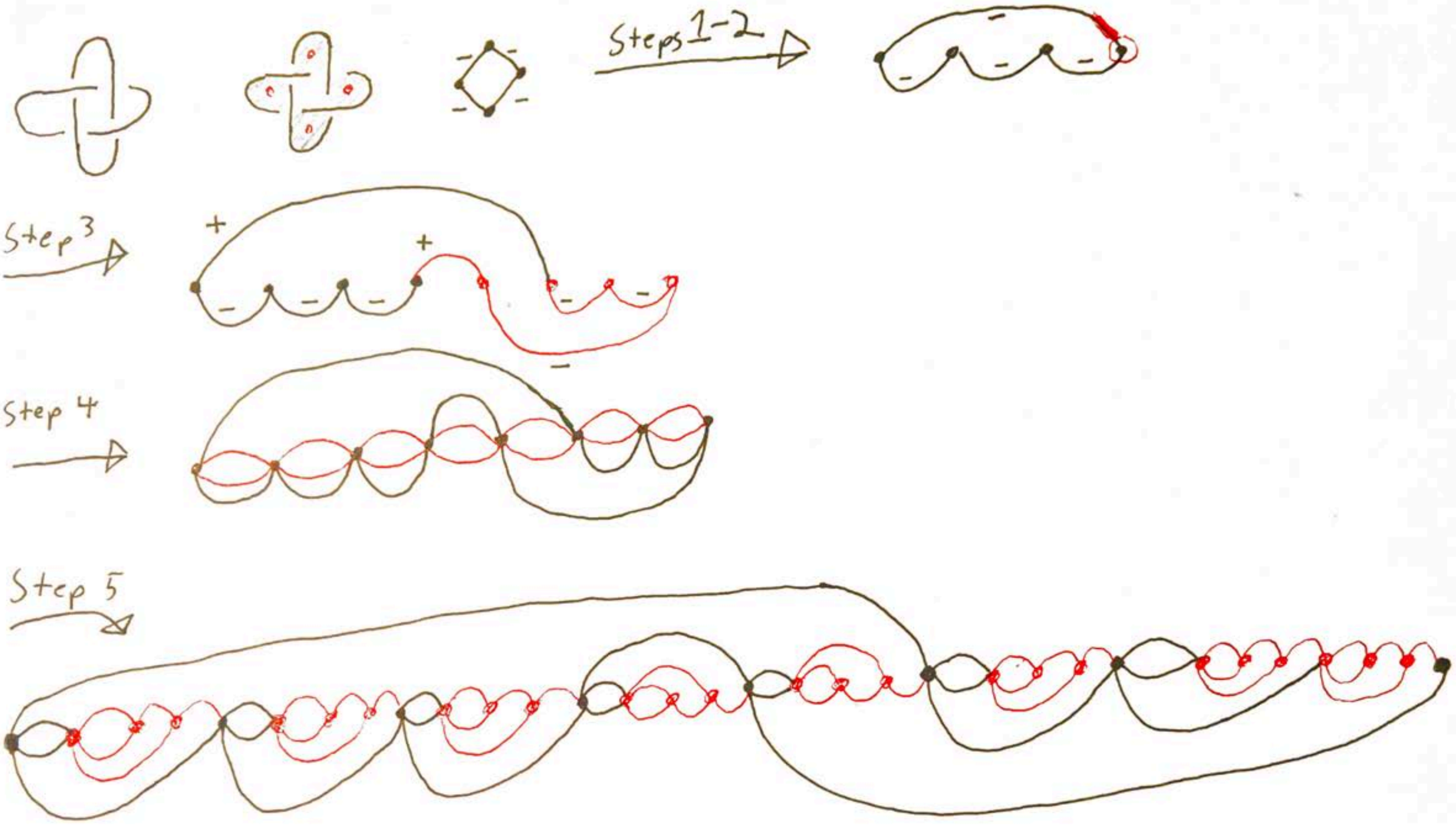
Now all edges have the correct sign.

Step 4: Ensure each vertex has an incident edge to the left: (except the leftmost vertex)

Step 5: Ensure that each vertex has <u>only one</u> incident edge to the left:



Now each non-root vertex has exactly one left-neighbor, its parent. □



Steps 1-2
Step 3
Step 4
Step 5